①

Integral calculus

Indefinite Integral

Definite Integral

$$\frac{d}{dx}\left[F(x)\right] = f(x) \therefore \int f(x)\,dx = F(x)+c$$

1. $$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{\cancel{n+1}}(\cancel{n+1})\,x^{n}$$

$$\therefore \int x^{n}\,dx = \frac{x^{n+1}}{n+1}+c \qquad n \neq -1$$

2. $$\frac{d}{dx}(\sin x) = \cos x \therefore \int \cos x\,dx = \sin x + c$$

3. $$\frac{d}{dx}(\cos x) = -\sin x$$

$$\therefore \int \sin x\,dx = -\cos x + K$$

4. $$\frac{d}{dx}(\tan x) = \sec^{2} x$$

$$\therefore \int \sec^{2} x\,dx = \tan x + c$$

5. $$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^{2} x$$

$$\therefore \int \operatorname{cosec}^{2} x\,dx = -\cot x + c$$

6. $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\int \sec x \tan x \, dx = \sec x + K$

7. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\therefore \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + K$

8. $\frac{d}{dx}(\log_e x) = \frac{1}{x} \quad \therefore \int \frac{1}{x} dx = \log_e x + c$

9. $\frac{d}{dx}(e^x) = e^x \quad \therefore \int e^x dx = e^x + K$

10. $\frac{d}{dx}(a^x) = a^x \log_e a$

$\int a^x dx = \frac{a^x}{\log_e a} + K$

11. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\therefore \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x.$

Integration by substitution.

Type I: $\int \frac{f'(x)}{f(x)} dx$

Put $f(x) = t$

$f'(x) = \frac{dt}{dx}$

$\therefore f'(x)dx = dt$

$= \int \frac{dt}{t} = \log t + K = \log[f(x)] + K$

For Exam:

$\int \tan x\,dx$; $\int \cot x\,dx$; $\int \sec x\,dx$;

$\int \operatorname{cosec} x\,dx$

1. $\int \tan x\,dx = \int \frac{\sin x}{\cos x} dx$

Put $\cos x = t$

$-\sin x\,dx = dt$

$\therefore \sin x\,dx = -dt$

$= -\int \frac{dt}{t} = -\log t + K = -\log \cos x + K$

or $\log \sec x + K$

2. $\int \frac{x^4}{1+x^5} dx$

Put $1 + x^5 = t$

$5x^4\,dx = dt$

$x^4\,dx = \frac{dt}{5}$

$= \frac{1}{5}\int \frac{dt}{t} = \frac{1}{5}\log t + K$

$= \frac{1}{5}\log(1+x^5) + K$

Type – II: $\int f'(x) \cdot f(x)\, dx$

For Exam:

1. $\int e^{\tan x} \sec^2 x\, dx$ Put $\tan x = t$

$\sec^2 x\, dx = dt$

$= \int e^t dt = e^t + c = e^{\tan x} + c$

2. $\int \frac{(4 + 5\tan^{-1} x)}{1 + x^2} dx$; 3. $\int \frac{dx}{x(1 + \log x)}$

4. $\int \frac{1}{\sqrt{x}} \sin \sqrt{x}\, dx$ - - - .

$\sqrt{a^2 - x^2}$; Put $x = a\sin\theta$ or $a\cos\theta$

$\sqrt{a^2 + x^2}$; Put $x = a\tan\theta$ or $a\cot\theta$

$\sqrt{x^2 - a^2}$; Put $x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$

$$\left[\begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \sec^2\theta = 1 + \tan^2\theta \\ \mathrm{cosec}^2\theta = 1 + \cot^2\theta \end{array}\right]$$

For Exam: $\int \frac{x^4\,dx}{\sqrt{a^2-x^2}}$ Put $x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

$$= \int \frac{a^4 \sin^4\theta \times a\cos\theta\,d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} = \int \frac{a^4 \sin^4\theta \times \cancel{a\cos\theta}}{\cancel{a\cos\theta}}\,d\theta$$

$$= a^4\int \sin^4\theta\,d\theta = a^4 \int \left[\frac{1-\cos 2\theta}{2}\right]^2 d\theta$$

$$= \frac{a^4}{4}\int (1+\cos^2 2\theta - 2\cos 2\theta)\,d\theta$$

$$= \frac{a^4}{4}\int \left[1 + \frac{1+\cos 4\theta}{2} - 2\cos 2\theta\right] d\theta$$

$$= \frac{a^4}{4}\int d\theta + \frac{a^4}{8}\int (1+\cos 4\theta)\,d\theta - \frac{a^4}{2}\int \cos 2\theta\,d\theta$$

$$\left[\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)\right]$$

$$= \frac{a^4}{4}[\theta] + \frac{a^4}{8}\left\{\theta + \frac{\sin 4\theta}{4}\right\} - \frac{a^4}{2}\left[\frac{\sin 2\theta}{2}\right] + k$$

where $\theta = \sin^{-1} x/a$