7.4-1 Phase

The position of any alternating quantity which shows that part is to be increased forward from the cycle or time period from the reference point, is called **phase.** It can be move clearly fig. 7.11. The phase of point A of alternating quantity on taking reference point O is $\frac{\pi}{2}$ or $\frac{T}{4}$ and phase of piont B is π or $\frac{T}{2}$.

7.4-2 Phase Difference

According to fig. 7.17 it assumed that two same and one turn wound coils are displaced at α° from each other and both are revolving in any magnetic field of uniform strength. The value induced e.m.f. in both coils will be equal in this direction but their maximum and minimum values will not received in a time but the maximum value in coil A and minimum value will displaced at α° from the coil B. This displacement is called phase difference because in upper position the maximum value of induced e.m.f. gets primary in coil A and secondary in coil B. Hence, induced e.m.f. in coil A is called **leading** from the e.m.f. induced in coil B and its opposite the induced e.m.f. in coil B is called **lagging** from the e.m.f. induced in coil A.

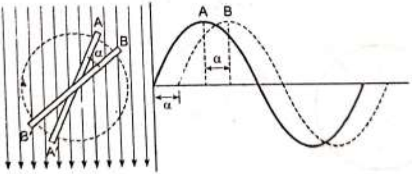

Fig. 7.17

The angle of displacement between these both e.m.f. is called angle of lead and angle of lag.

The means of leading alternating quantity is that quantity which gets maximum value, earlier compare to other quantity.

According to fig. 7.17 leading e.m.f. A flows first from zero and maximum values and e.m.f. B flows through zero and maximum values after α° periodic angle. Hence, B is lagging angle α from A. So, if A is assumed as reference then value of instantaneous e.m.f. will be determined by

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - \alpha)$$

Hence, according to fig. 7.18 quantity B is leading **angle** α from quantity B, so its equation will be as follows

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin (\omega t + \alpha)$$

When (+) positive sign is used with phase difference then it indicates lead and when (−) negative sign is used with phase difference then it indicates lag. In fig. 7.18 (a) graphic representation and in (b) vector representation of both voltages are shown.

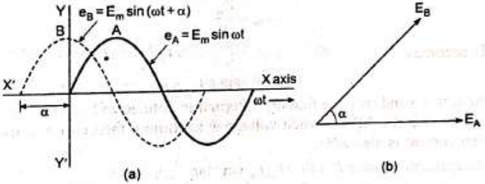

Fig. 7.18

§7.5 CIRCUITS CONTAINING ONLY PURE RESISTANCE

A pure resistive circuit is that circuit which have not inductance and capacitance so when current changes in circuit then back e.m.f. does not produced. Hence, total applied voltage will be supply ohmic voltage drop only as perform in D.C. circuits. Hence, we use the effective value. Means $I = \frac{V}{R}$ (In phase voltage V) so power at any instant in circuit is the product of instantaneous voltage and instantaneous current $P = V \times i$

Consider on circuit shown in fig. 7.19. Suppose, applied voltage is represent by following equation

$$V = V_m \sin \theta = V_m \sin \omega t \quad \text{...(i)}$$

Suppose, R is ohmic resistance and i = Instantaneous current. Applied voltage will be equal voltage drop in resistance of circuit.

$$V = iR \quad \text{...(ii)}$$

Putting the value of V in Eq. (ii) from Eq. (i), we get

$$V_m \sin \omega t = iR$$

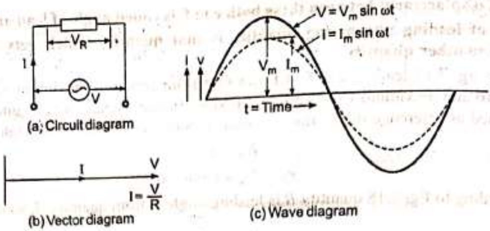

Fig. 7.19

$$\therefore \qquad i = \frac{V_m \sin \omega t}{R} \qquad \text{...(iii)}$$

The value of current i will be maximum at that time when value of $\sin \omega t$ will be maximum (when $\sin \omega t$ is unity)

$$I_m = \frac{V_m}{R}$$

Now, Eq. (iii) becomes

$$i = I_m \sin \omega t \qquad \text{...(iv)}$$

Comparing the Eqs. (i) and (iv), we find that alternating voltage and current are in phase with each other as shown in fig. 7.19 (b). Means when voltage is maximum then currents also maximum when voltage is zero then current is also zero.

Power : Instantaneous power $P = Vi = V_m I_m \sin^2 \omega t$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$P = \frac{1}{2} V_m I_m - \frac{1}{2} V_m I_m \cos 2\omega t \qquad \text{...(v)}$$

It is clear from Eq. (v). The constant part of power is $\frac{1}{2} V_m I_m$ and fluctuating part is $-\frac{1}{2} V_m I_m \cos 2\omega t$.

Average value of other fluctuating part is zero for a complete cycle. Hence, power of any circuit which is measured by wattmeter is a average of instantaneous power (p)

$$P = \text{Average of } p$$

$$= \text{Constant part of } p$$

$$P = \frac{1}{2} V_m I_m = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V \cdot I \text{ W}$$

Here, V = r.m.s. value of applied voltage

and I = r.m.s. value of current

voltage drop V_R and BA indicates inductive drop. Applied voltage OA will be sum of vectors OB and BA

$$V^2 = (V_R)^2 + (V_L)^2$$
$$V^2 = (IR)^2 + (IX_L)^2$$
$$V = I\sqrt{R^2 + X_L^2}$$

or
$$\frac{V}{I} = \sqrt{R^2 + X_L^2}$$

Impedance : Total resultant resistance $\sqrt{R^2 + X_L^2}$ of circuit is **denoted by word Z** and this is called **impedance** and is **measured in ohm.**

Hence
$$\frac{V}{I} = Z$$

or
$$I = \frac{V}{Z} \qquad \left[\text{while } I = \frac{V}{R} \text{ in D.C. circuits}\right]$$

$$\frac{VR}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$\frac{R}{Z}$ is called **power factor of circuit.** Impedance triangle can be made by voltage triangle. Impedance triangle is shown in fig. 7.29. It is clear from fig. 7.28 (b). Applied voltage V is leading angle ϕ from current I or current I is lagging angle ϕ from voltage V.

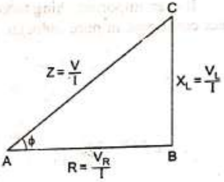

Fig. 7.29 Impedence

From impedance triangle ABC

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$

$$\therefore \qquad \phi = \tan^{-1}\frac{X_L}{R} \text{ and } \cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$$

$$\phi = \cos^{-1}\frac{R}{Z}$$

So, we can determined from Impedance triangle ABC

$$Z^2 = R^2 + X_L^2$$

$$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Reactance})^2$$

$$\textbf{Impedance } Z = \sqrt{R^2 + X_L^2}$$

The relaton as above mentioned is true for every circuit.

Power : If applied voltage is denoted by equation $V = V_m \sin\omega t$ and current by equation $i = I_m \sin(\omega t - \phi)$

Then
$$\text{Instantaneous power } P = V \times i$$

or
$$P = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

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$$P = V_m I_m \sin \omega t \sin (\omega t - \phi)$$

$$P = V_m I_m \left[\frac{\cos \phi - \cos (\omega t - \phi)}{2}\right]$$

$$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} I_m V_m \cos (2\omega t - \phi) \qquad [\text{Average of } \cos (2\omega t - \phi) = 0]$$

Instantaneous Average of power $\beta = \frac{1}{2} V_m I_m \cos \phi$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

$$= V_{rms} \times I_{rms} \times \cos \phi$$

Power (P) = r.m.s. value of voltage × r.m.s. value of current × power factor

$\cos \phi$ is called **power factor.** The product of r.m.s. value of voltage and r.m.s. value of current is **called apparent power. It is measured by volt × ampere while $VI \cos \phi$ is called the power. The ratio of true power and apparent power is called power factor.**

True power (W) = volt × *ampere* × *power factor*

Watt $= V \cdot I \cos \phi$

It is an important thing to keep in mind that power is only consumed in Ohmic resistance and does not consumed in pure inductance.

$$P = V \cdot I \cos \phi = V \cdot I \frac{R}{Z} \qquad \left(\because \cos \phi = \frac{R}{Z}\right)$$

$$P = \frac{V \cdot I}{Z}, \quad R = I^2 R \qquad \left(\because \frac{V}{Z} = I\right)$$

$$P = I^2 R$$

7.6-1 Apparent Power, True Power and Reactive Power

As we have read that if there is pure resistive circuit then current will be in phase of voltage. If circuit is pure inductive then current lagging from voltage V is $\frac{\pi}{2}$ or leading 90° and when circuit is pure capacitive then current is $\frac{\pi}{2}$ or 90° from voltage. If circuit is compound then current can be in phase of applied voltage or lagging from the voltage or leading from the voltage.

Assume any series circuit which has inductive qualities and current is lagging from voltage. Then curent will be to components :

(i) Active component : The component of current which is in phase of applied voltage is called **active component** means $I \cos \phi$. It is also called watt full component.

(ii) Reactive component : The component of current which is in quadrature of field of voltage is also called **reactive component** as $I \sin \theta$. It is also called wattless or idle component.

(a) Apparent Power : The product of r.m.s. value of current and voltage in alternating current circuit is called **apparent power.** It units is volt-ampere (VA) and its large unit is kilovolt ampere.

(b) **True power :** The product of apparent power and power factor is called true power in alternating circuit. It is denoted by $V \cdot I \cos\phi = w$. **Its unit is watt or kilowatt.**

Reactive power : The product of apparent power ($V \cdot I$) and $\sin\phi$ of phase angle between current and voltage ($V \cdot I \sin\phi$) is called reactive power. Its unit is VAR or kVAR. By product of $\sin\phi$ from VA or kVA. Reactive power can be calculated.

Above relation can be clear from the kVA.

Triangle given in fig. 7.30

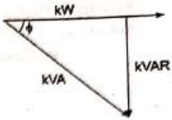


Fig. 7.30

$$\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2}$$

$$\text{kW} = \text{kVA} \cos\phi$$

and

$$\textbf{kVAR} = \textbf{kVA} \sin\phi$$

Note : Power consumed in pure inductive and pure capacitive circuit is to be zero means power loss in this type of circuits is only due to resistance.

7.6-2 Power Factor

Power factor can be defined in single phase system or in balance load three phase system of A.C. current.

1. Cosine of angle between voltage and current lagging or leading.
2. Ratio of resistance and impedance $= \dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}} = \cos\phi$
3. Ratio between true power and apparent power $= \dfrac{W}{V \cdot I} = \cos\phi$

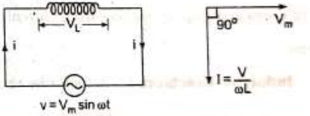
The value of power factor is unity in pure resistive circuit.

The value of power factor is zero in pure inductive and pure capacitive circuit.

7.5-1 Circuit containing only pure inductance

A pure inductive circuit is that circuit which has only inductance but there is not resistance and capacitance. A coil made of thick insulated copper wire on **laminated iron core** can be assumed as pure **inductive coil.** It is also be called **choke coil.**

Magnetic field produced by alternating current is alternating so its magnitude will vary at every instant. Whenever magnetic flux linked in any circuit changes then self induced e.m.f. will be produced in coil due to this flux. Because the resistance of coil is negligible. So, applied voltage has to overcome this self induced e.m.f. only. So, the applied e.m.f. at every step is about to equal or opposite to self induced e.m.f.



(a) Circuit diagram (b) Vector diagram

Fig. 7.21

Suppose, Applied voltage $V = V_m \sin \omega t$

and Inductance of coil $= L$ henry

$$\therefore \quad v \text{ at every Instant} = -e' = +L\frac{di}{dt} \text{ (Formula)}$$

$$v = V_m \sin \omega t$$

$$V_m \sin \omega t = \frac{di}{dt} L \qquad \left(di = \frac{V_m}{L} \sin \omega t \, dt\right)$$

Integrating of both sides we will get the value of i

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right) = -\frac{V_m}{\omega L} \cos \omega t$$ Constant integration $= 0$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \text{ (Formula)}$$

The maximum value of $i = I_m = \frac{V_m}{\omega L}$ when $\sin\left(\omega t - \frac{\pi}{2}\right)$ will be unity.

Hence, equaion of current can be written as follows

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$ (Putting on $\frac{V_m}{\omega L} = I_m$)

Hence, we see that when applied voltage is represented by $V = V_m \sin \omega t$ then current i in pure inductive circuit is given by

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence, comparing the equations of instantaneous voltage and instantaneous current it is clear that current is lagging $\frac{\pi}{2}$ rad from the applied voltage. In other words, phase difference between both is $\frac{\pi}{2}$ rad, while voltage is leading from current. In fig. 7.21 (b) vector diagram of inductive circuits are shown.

Inductive Reactance : As describe above that $I_m = \frac{V_m}{\omega L}$

ωL acts as resistance in inductive circuits and it is called **inductive reactance** which is written in ohm. It is denoted by word X_L.

Mean $X_L = \omega L$ if L is in Henry and ω is in radian per second then X_L will be in ohm.

Power : Instantaneous power $P = vi$

If we put $v = V_m \sin \omega t$ and $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

$$i = -I_m \cos \omega t$$

The Instantaneous power $P = V_m \sin \omega t \times -I_m \cos \omega t$

$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

$\therefore$ **Average power for a complete cycle** $P = -\frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$

Hence, we get total power zero in these circuits. This as result which is **wonderful in primary** while it apparent that both V and I are finite, not zero. In actual this power wave is sinusoidal wave of pure double frequency whose maximum value $\frac{1}{2}V_m I_m = V \cdot I$.

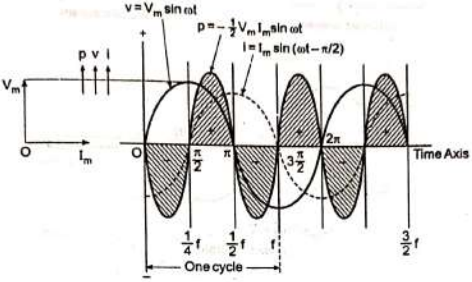


Fig. 7.22

The diagram of power wave is shown in fig. 7.22. It is clear from the figure average demand of power from the supply for a complete cycle is zero.

7.5-2 Circuit containing only pure capacitance

As we read that when any dielectric material is placed between two conductive plate switch are near to each other then one capacitor is formed. When D.C. voltage is applied across the plates of any capacitor then capacitor becomes charged quickly whenever voltage of capacitor becomes equal to the supply voltage at that time flow of electric current becomes stop. It shows that capacitor acts as a heavy resistance in current circuits but when capacitor is connected from A.C. supply the capacitor is charged first in one direction and then in opposite direction.

Suppose, a capacitor of C capacitance is connected across the alternating voltage just like fig. 7.23 (a). If the value of alternating voltage across the ends of capacitor at any instant is V and supply voltage is represented by following equation

$$v = V_m \sin \omega t$$

Then electric charge involved in capacitor at that instant

$$q = C \cdot v \quad \text{...(i)}$$

Putting the value of $V = V_m \sin \omega t$ in Eq. (i) because electric quantity q is a product of current and time, so if current is variable then

$$q = \int i\, dt$$

or

$$dq = i\, dt$$

$\therefore$

$$i = \frac{dq}{dt} = C\left(\frac{dv}{dt}\right) \qquad (\because q = Cv)$$

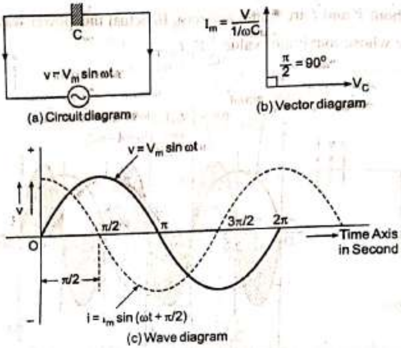


Fig. 7.23

$\therefore$ $$V = V_m \sin \omega t$$

$\therefore$ $$i = CV_m \omega \cos \omega t$$

$\therefore$ $$i = \frac{V_m}{1/\omega C} \cos \omega t$$

$$= \frac{V_m}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \left[\text{when, } \sin\left(\omega t + \frac{\pi}{2}\right) = 1\right]$$

$\therefore$ $$I_m = \frac{V_m}{1/\omega C}$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Capacitive Reactance : $\frac{1}{\omega C}$ is called capacitive reactance and it is denoted in ohm. (If C is in farad and ω is in radian per second)

We see that if applied voltage is represented by equation $V = V_m \sin \omega t$ then current will be represented by $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

Hence, comparing the equations of instantaneous voltage and instantaneous current it is clear that current is leading $\frac{\pi}{2}$ radian from applied voltage. In other words, phase difference between voltage and current is $\frac{\pi}{2}$ radian while current is leading from voltage as shown in fig. 7.23 (b).

Power : Instantaneous power $P = V \times i$