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Department of Civil Engineering do Diploma –

4th SEM

05- Lecture Notes on

R.C.C. T Beam

Based on limit method

T−beams and L−beams

Beams having effectively *T-sections* and *L-sections* (called *T*-beams and *L*-beams) are commonly encountered in beam-supported slab floor systems [Figs. 2.8]. In such situations, a portion of the slab acts integrally with the beam and bends in the longitudinal direction of the beam. This slab portion is called the *flange* of the T- or L-beam. The beam portion below the flange is often termed the *web*, although, technically, the web is the full rectangular portion of the beam other than the overhanging parts of the flange. Indeed, in shear calculations, the web is interpreted in this manner.

When the flange is relatively wide, the flexural compressive stress is not uniform over its width. The stress varies from a maximum in the web region to progressively lower values at points farther away from the web. In order to operate within the framework of the theory of flexure, which assumes a *uniform* stress distribution across the *width* of the section, it is necessary to define a *reduced effective flange*.

The _effective width of flange' may be defined as the width of a hypothetical flange that resists in-plane compressive stresses of uniform magnitude equal to the peak stress in the original wide flange, such that the value of the resultant longitudinal compressive force is the same (Fig. 2.8).

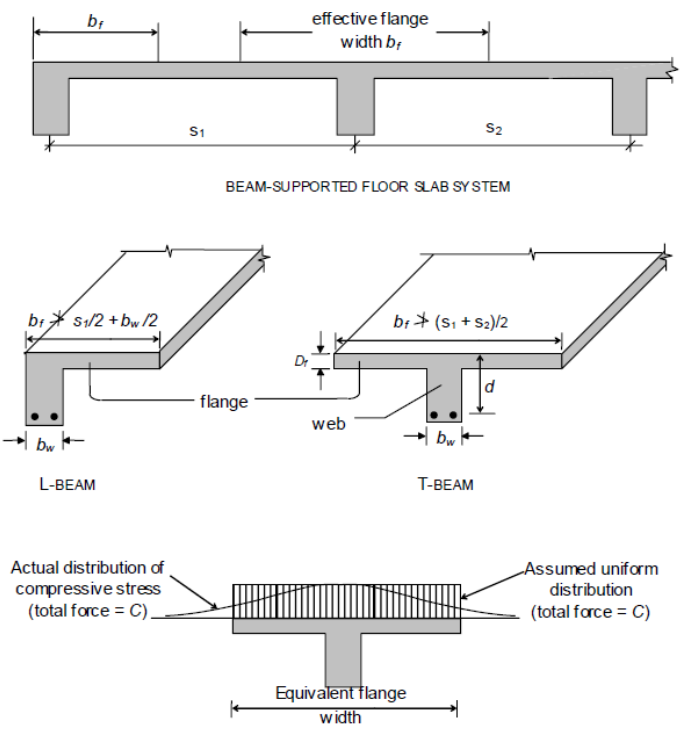

Figure 2.8 T-beams and L-beams in beam-supported floor slab systems

The effective flange width is found to increase with increased span, increased web width and increased flange thickness. It also depends on the type of loading (concentrated, distributed, etc.) and the support conditions (simply supported, continuous, etc.). Approximate formulae for estimating the ‗effective width of flange' b_f (Cl. 23.1.2 of Code) are given as follows:

$$b_f = \begin{cases} l_0/6 + b_w + 6D_f & for\ T-Beam \\ l_0/12 + b_w + 3D_f & for\ L-Beam \end{cases} \quad (12)$$

where b_w is the breadth of the web, D_f is the thickness of the flange [Fig 2.8], and l_0 is the ―distance between points of zero moments in the beam‖ (which may be assumed as 0.7 times the effective span in continuous beams and frames). Obviously, b_f cannot extend beyond the slab portion tributary to a beam, i.e., the actual width of slab available. Hence, the calculated b_f should be restricted to a value that does not exceed $(s_1 + s_2)/2$ in the case of T−beams, and $s_1/2 + b_w/2$ in the case of L−beams, where the spans s_1 and s_2 of the slab are as marked in Fig. 2.8.

In some situations, *isolated* T−beams and L−beams are encountered, i.e., the slab is discontinuous at the sides, as in a footbridge or a ‗stringer beam' of a staircase. In such cases, the Code [Cl. 23.1.2(c)] recommends the use of the following formula to estimate the ‗effective width of flange' b_f:

$$b_f = \begin{cases} \dfrac{l_0}{l_0/b+4} + b_w & for\ isolated\ T-Beams \\ \dfrac{0.5 l_0}{l_0/b+4} + b_w & for\ isolated\ L-Beam \end{cases} \quad (13)$$

where b denotes the *actual* width of flange; evidently, the calculated value of b_f should not exceed b.

Analysis of Singly Reinforced Flanged Sections

The procedure for analysing flanged beams at ultimate loads depends on whether the neutral axis is located in the flange region [Fig. 2.8(a)] or in the web region [Fig. 2.8(b)].

If the neutral axis lies within the flange (i.e., $x_u \le D_f$), then as in the analysis at service loads all the concrete on the tension side of the neutral axis is assumed ineffective, and the T-section may be analysed as a rectangular section of width b_f and effective depth d [Fig. 2.8(a)]. Accordingly, Eq. (7) and Eq. (9) are applicable with b replaced by b_f.

If the neutral axis lies in the web region (i.e., $x_u > D_f$), then the compressive stress is carried by the concrete in the flange and a portion of the web, as shown in Fig. 2.8(b). It is convenient to consider the contributions to the resultant compressive force C_u, from the *web* portion ($b_w \times x_u$) and the *flange* portion (width $b_f - b_w$) separately, and to sum up these effects. Estimating the compressive force C_{uw} in the ‗web' and its moment contribution M_{uw} is easy, as the full stress block is operative:

$$C_{uw} = 0.361\ f_{ck} b_w x_u \tag{14}$$

$$M_{uw} = C_{uw}(d - 0.416 x_u) \tag{15}$$

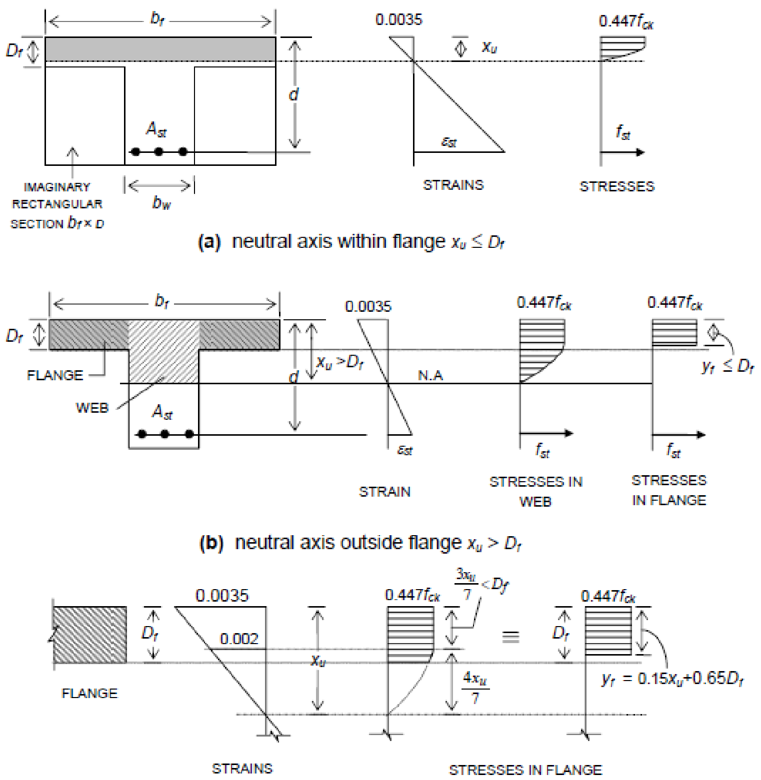

Figure 2.9 Behaviour of flanged beam section at ultimate limit state

However, estimating the compressive force C_{uf} in the flange is rendered difficult by the fact that the stress block for the flange portions may comprise a rectangular area plus a truncated parabolic area [Fig. 2.8(b)]. A general expression for the total area of the stress block operative in the flange, as well as an expression for the centroidal location of the stress block, is evidently not convenient to derive for such a case. However, when the stress block over the flange depth contains only a rectangular area (having a uniform stress 0.447 f_{ck}), which occurs when $\frac{3}{7}x_u \geq D_f$, an expression for C_{uf} and its moment contribution M_{uf} can easily be formulated. For the case, $1 < x_u / D_f < 7/3$, an *equivalent* rectangular stress block (of area $0.447 f_{ck} y_f$) can be conceived, for convenience, with an equivalent depth $y_f \leq D_f$, as shown in Fig. 2.8(c). The expression for y_f given in the Code (Cl. G − 2.2.1) is necessarily an approximation, because it cannot satisfy the *two* conditions of 'equivalence', in terms of area of stress block as well as centroidal location. A general expression for y_f may be specified for any $x_u > D_f$:

$$y_f = \begin{cases} 0.15x_u + 0.65D_f & for 1 < x_u / D_f < 7/3 \\ D_f & for\ D_f \geq 7/3 \end{cases} \tag{16}$$

The expressions for C_{uf} and M_{uf} are accordingly obtained as:

$$C_{uf} = 0.447 f_{ck} (b_f - b_w) y_f \quad for\ x_u > D_f \tag{17}$$

$$M_{uf} = C_{uf} (d - y_f / 2) \tag{17a}$$

The location of the neutral axis is fixed by the force equilibrium condition (with y_f expressed in terms of x_u [Eq. 17]).

$$C_{uf} + C_{uf} = f_{st} A_{st} \tag{18}$$

where $f_{st} = 0.87 f_y$ for $x_u \leq x_{u,max}$. Where $x_u > x_{u,max}$, the strain compatibility method has to be employed to determine x_u.

Substituting Eq. 14 and Eq. 17 in Eq. 18, and solving for x_u,

$$x_u = \frac{f_{st} A_{st} - 0.447 f_{ck} (b_f - b_w) y_f}{0.361 f_{ck} b_w} \quad for\ x_u > D_f \tag{19}$$

The final expression for the ultimate moment of resistance M_{uR} is obtained as:

$$M_{uR} = M_{uw} + M_{uf} \tag{20}$$

$$\Rightarrow M_{uR} = 0.361 f_{ck} b_w x_u (d - 0.416 x_u) + 0.447 f_{ck} (b_f - b_w) y_f (d - y_f / 2) \tag{21}$$

Limiting Moment of Resistance

The limiting moment of resistance $M_{u,lim}$ is obtained for the condition $x_u = x_{u,max}$, where $x_{u,max}$ takes the values of 0.531*d*, 0.479*d* and 0.456*d* for Fe 250, Fe 415 and Fe 500 grades of tensile steel reinforcement. The condition $x_u/D_f \geq 7/3$ in Eq. 4.69, for the typical case of Fe 415, works out, for $x_u = x_{u,max}$, as $0.479d/D_f \geq 7/3$, i.e., $Ddf \leq 0205$.. The Code (Cl. G−2.2) suggests a simplified condition of $d/D_f \leq 0.2$ for *all* grades of steel — to represent the condition $x_u/D_f \geq 7/3$.

Eq. (21) and Eq. (16) take the following forms:

$$M_{u,\lim} = 0.361 f_{ck} b_w x_{u,\max}(d - 0.416 x_{u,\max}) + 0.447 f_{ck}(b_f - b_w) y_f (d - y_f/2) \text{ for } x_{u,\max} > D_f \tag{22}$$

$$y_f = \begin{cases} 0.15 x_{u,\max} + 0.65 D_f & \text{for } D_f/d > 0.2 \\ D_f & \text{for } D_f/d \leq 0.2 \end{cases} \tag{23}$$

The advantage of using Eq. (23) in lieu of the more exact Eq. (16) (with $x_u = x_{u,max}$) is that the estimation of y_f is made somewhat simpler. Of course, for $x_{u,max} \leq D_f$ (i.e., neutral axis within the flange),

$$M_{u,\lim} = 0.361 f_{ck} b_f x_{u,\max}(d - 0.416 x_{u,\max}) \text{ for } x_{u,\max} \leq D_f \tag{24}$$

As mentioned earlier, when it is found by *analysis* of a given T-section that $x_u > x_{u,max}$, then the strain compatibility method has to be applied. As an approximate and conservative estimate, M_{uR} may be taken as $M_{u,lim}$, given by Eq. (23) / (24). From the point of view of *design* (to be discussed in Chapter 5), $M_{u,lim}$ provides a measure of the ultimate moment capacity that can be expected from a T-section of given proportions. If the section has to be designed for a *factored moment* $M_u > M_{u,lim}$, then this calls for the provision of compression reinforcement in addition to extra tension reinforcement.

Design Procedure

In the case of a *continuous* flanged beam, the negative moment at the face of the support generally exceeds the maximum positive moment (at or near the midspan), and hence governs the proportioning of the beam cross-section. In such cases of negative moment, if the slab is located on top of the beam (as is usually the case), the flange is under flexural tension and hence the concrete in the flange is rendered ineffective. The beam section at the support is therefore to be designed as a rectangular section for the factored negative moment. Towards the midspan of the beam, however, the beam behaves as a proper flanged beam (with the flange under flexural compression). As the width of the web b_w and the overall depth D are already fixed from design considerations at the support, all that remains to be determined is the area of reinforcing steel; the *effective width of flange* is determined as suggested by the Code .

The determination of the actual reinforcement in a flanged beam depends on the location of the neutral axis x_u, which, of course, should be limited to $x_{u,max}$. If M_u exceeds $M_{u,lim}$ for a singly reinforced flange section, the depth of the section should be suitably increased; otherwise, a doubly reinforced section is to be designed.

Neutral Axis within Flange ($x_u \le D_f$):

This is, by far, the most common situation encountered in building design. Because of the very large compressive concrete area contributed by the flange in T-beam and L-beams of usual proportions, the neutral axis lies within the flange ($x_u \le D_f$), whereby the section behaves like a rectangular section having width b_f and effective depth d.

A simple way of first checking $x_u \le D_f$ is by verifying $M_u \le (M_{uR})_{x_u=D_f}$ where $(M_{uR})_{x_u=D_f}$ is the limiting ultimate moment of resistance for the condition $x_u = D_f$ and is given by

$$(M_{uR})_{x_u=D_f} = 0.361\, f_{ck} b_f D_f (d - 0.416 D_f) \tag{25}$$

It may be noted that the above equation is meaning only if $x_{u,\max} > D_f$. In rare situations involving very thick flanges and relatively shallow beams, $x_{u,\max}$ may be less than D_f. in such cases, $M_{u,lim}$ is obtained by substituting $x_{u,max}$ in place of D_f in Eq. (25).

***Neutral Axis within Web* ($x_u > D_f$):**

When $M_u > (M_{uR})_{x_u=D_f}$, it follows that $x_u > D_f$. The accurate determination of x_u can be laborious. The contributions of the compressive forces C_{uw} and C_{uf} in the ‗web' and ‗flange' may be accounted for separately as follows:

$$M_{uR} = C_{uw}(d - 0.416 x_u) + C_{uf}(d - y_f / 2) \tag{26}$$

$C_{uw} = 0.361\, f_{ck} b_w x_u$ (27)

$C_{uf} = 0.447 f_{ck} (b_f - b_w)\, y_f$ (28)

and the equivalent flange thickness y_f is equal to or less than D_f depending on whether x_u exceeds $7D_f/3$ or not.

For $x_{u,max} \geq 7D_f/3$, the value of the ultimate moment of resistance $(M_{uR})_{x_u = 7D_f/3}$ corresponding to $x_u = 7D_f/3$ and $y_f = D_f$ may be first computed. If the factored moment $M_u \geq (M_{uR})_{x_u = 7D_f/3}$, it follows that $x_u > 7D_f/3$ and $y_f = D_f$. Otherwise, $D_f < x_u > 7D_f/3$ for $(M_{uR})_{x_u = D_f} < M_u < (M_{uR})_{x_u = 7D_f/3}$ and

$y_f = 0.15 x_u + 0.65 D_f$ (29)

Inserting the appropriate value — Df or the expression for y_f in Eq. (29), in Eq. (26), the resulting quadratic equation (in terms of the unknown x_u) can be solved to yield the correct value of x_u. Corresponding to this value of x_u, the values of C_{uw} and C_{uf} can be computed [Eq. (27), (28)] and the required A_{st} obtained by solving the force equilibrium equation.

$T_u = 0.87\, f_f A_{st} = C_{uw} + C_{uf}$

$$\Rightarrow (A)_{st\ required} = \frac{C_{uw} + C_{uf}}{0.87\, f_y} \quad (30)$$

Numerical Problem

Q-6 A continuous T-beam has the cross-sectional dimensions shown in figure below. The web dimensions have been determined from the consideration of negative moment at support and shear strength requirements. The span is 10 m and the design moment at midspan under factored loads is 800 kNm. Determine the flexural reinforcement requirement at midspan. Consider Fe 415 steel. Assume that the beam is subjected to moderate exposure conditions.

Solution

Determining approximate A_{st}

Effective flange width b_f

Actual flange width provided =1500mm; D_f=100 mm; b_w=300mm

Maximum width permitted =(0.7x10000)/6 + 300 + (6x100) =2067 mm >1500 mm

Therefore, b_f =1500 mm

Assuming d=650 mm and a lever arm z equal to larger of 0.9d = 585 mm

And d- Df/2 = 600mm i.e. z=600 mm

$$(A_{st})_{required} = \frac{800x10^6}{0.87x415x600} = 3693 mm^2$$

- Providing 4 bars, $\phi_{reqd} = \sqrt{\dfrac{3693/4}{\pi/4}}$ = 34.3 mm, i.e., 36 mm.

  As 4–36ϕ bars cannot be accommodated in one layer within the width b_w = 300 mm, two layers are required.
  Assuming a reduced $d \approx$ 625 mm, $z \approx 625 - 100/2 = 575$ mm.

  $\Rightarrow (A_{st})_{reqd} \approx 3693 \times \dfrac{600}{575} = 3854 \text{ mm}^2.$

- Provide 5–32ϕ [A_{st} = 804 × 5 = 4020 mm^2] with 3 bars in the lower layer plus 2 bars in the upper layer, with a clear vertical separation of 32 mm — as shown in Fig. 5.11(b). Assuming 8 mm stirrups and a clear 32 mm cover to *stirrups*,

  $\Rightarrow d = 700 - 32 - 8 - \dfrac{1}{5}[(3 \times 16) + 2 \times (32 + 32 + 16)]$

  $= 700 - 40 - 41.6 = 618$ mm

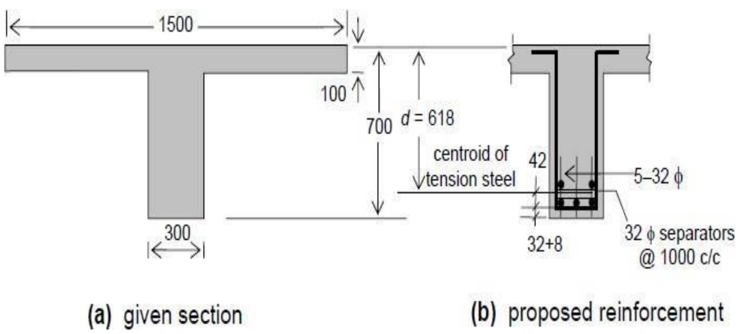

Figure: Reinforcement of T-beam of Example Problem

Determining actual Ast

$x_{u,max}$ = 0.479 × 618 =296 mm

As $x_{u,max} > D_f = 100mm$, the condition $x_u = D_f$ satisfies $\quad x_u \le x_{u,\max}$

• Assuming M 25 concrete, f_{ck} = 25 MPa

$(M_{uR})_{x_u = D_f}$ = 0.362 × 25 × 1500 × 100 × (618 – 0.416 × 100)

$= 782.5 \times 10^6$ Nmm < M_u = 800 kNm

$\Rightarrow x_u > D_f$ and $M_u = C_{uw}(d - 0.416\, x_u) + C_{uf}(d - y_f/2)$

where $C_{uw} = 0.362 f_{ck}\, b_w x_u$ = 0.362 × 25 × 300x_u = (2715x_u)N

and $C_{uf} = 0.447 f_{ck}(b_f - b_w)y_f$ = 0.447 × 25 × (1500 – 300)y_f = (13410y_f)

Considering $x_u = 7D_f/3$ = 233 mm (< $x_{u,max}$ = 296 mm), $y_f = D_f$ = 100 mm

$\Rightarrow (M_{uR})_{x_u = 7D_f/3} = (2715 \times 233)(618 - 0.416 \times 233) + (13410x100)x(618\text{-}100/2)$

$= 1091.3x10^6$ Nmm $> M_u = 800$ KNm

Evidently, $D_f < x_u < \frac{7}{3} D_f$, for which $y_f = 0.15x_u + o.65D_f$

$C_{uf} = 13410(0.15x_u.65 \times 100) = (2011.5x_u + 871650)$ N

$M_u = 800 \times 10^6 = (2715x_u)(618 - 0.416x_u)$

$+ (2011.5xu + 871650)x[618-(0.15xu+65)/2]$

$= -1280.3x_u^2 + 2790229.5\,x_u + 510.35 \times 106$

Solving this quadratic equation,

$\boldsymbol{x_u} = \mathbf{109.3}$ **mm** $< x_{u,max} = 296$ m

$\Rightarrow y_f = 0.15x_u + 65 = 81.4$ mm

Applying $T_u = 0.87 f_y A_{st} = C_{uw} + C_{uf}$

$$(A)_{st\ required} = \frac{(2715x109.3) + (13410x81.4)}{0.87x415} = 3845\ mm^2$$

The reinforcement (5-32Φ; A_{st}=4020 mm^2, based on appropriate estimate of A_{st} [Fig.] is evidently adequate and appropriate.

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